

Comments on interactions in the SUSY models

Sudhaker Upadhyay^a, Alexander Reshetnyak^b, and Bhabani Prasad Mandal^a

^a*Department of Physics, Banaras Hindu University, Varanasi-221005, India*

^b*Institute of Strength Physics and Materials Science of SB RAS, 634021, Tomsk, Russia*

We consider special supersymmetry (SUSY) transformations with m generators \overleftarrow{s}_α for a certain class of models and study some physical consequences of Grassmann-odd transformations which form an Abelian supergroup with finite parameters and respective group-like elements being functionals of field variables. The SUSY-invariant path integral measure within conventional quantization implies the appearance, under a change of variables related to such SUSY transformations, of a Jacobian which is explicitly calculated. The Jacobian implies, first of all, the appearance of trivial interactions in the transformed action, and, second, the presence of a modified Ward identity which reduces to the standard Ward identities in the case of constant parameters. We examine the case of $N = 1$ and $N = 2$ supersymmetric harmonic oscillators to illustrate the general concept by a simple free model with $(1, 1)$ physical degrees of freedom. It is shown that the interaction terms U_{tr} have a corresponding SUSY-exact form: $U_{tr} = (V_{(1)} \overleftarrow{s}; V_{(2)} \overleftarrow{s} \overleftarrow{s})$ naturally generated in this generalized formulation. We argue that the case of non-trivial interactions cannot be obtained in such a way.

I. INTRODUCTION

Supersymmetric theories are invariant under SUSY transformations which relate the bosonic and fermionic degrees of freedom present in the theories and were initially proposed with the motivation of studying the fundamental interactions in a unified manner. The generators of SUSY transformations satisfy Lie superalgebra relations which are closed under the combination of commutators and anticommutators. Local or nonlinear versions of the Lie superalgebra construction were extended to various field-theoretic models, such as superstring theories [1], supergravity [2, 3] (for modern developments, see Refs. [4, 5]) and higher-spin field theories [6–11]. SUSY theories provide a bosonic superpartner to each fermion present in a theory and vice-versa. This indicates that if $N = 1$ SUSY (with one fermionic generator in terms of Dirac spinor) is to be a perfect symmetry of nature, then each set of superpartners must have the same set of quantum numbers with the only difference in spin. Despite the beauty of all such unified theories, SUSY has not been supported by experimental evidence so far, but remains one of the problems included in LHC experimental program.

Some variants of SUSY have also provided an interesting topic in quantum mechanics [12] due to a link to exactly solvable models. SUSY and its breaking have been studied in various simple quantum mechanical systems involving a spin-1/2 particle moving in one direction [13, 14]. The supersymmetric Hamiltonian may be presented in terms of supercharges which generate SUSY transformations. A path integral formulation of SUSY in quantum mechanics was first analysed by Salomonson and van Holten [15]. Further, by using SUSY methods, the rate of tunnelling through quantum mechanical barriers was accurately determined [16–19].

SUSY transformations, when applied to gauge theory, together with special global SUSY transformations known as BRST transformations [20, 21], have also been explored in a more effective way [22, 23]. The BRST symmetry and the associated concept of BRST cohomology provide the commonly used quantization methods in Lagrangian [24, 25] and Hamiltonian [26, 27] formalisms for gauge and string theories [28, 29]. The BRST symmetry was generalized [25] to the case of an infinitesimal field-dependent (FD) transformation parameter μ , $\mu^2 = 0$, within the field-antifield formalism [24, 25] in order to prove the independence from small gauge variations of the path integral for arbitrary gauge theories. A further generalization [30] was made in Yang–Mills theories with R_ξ -gauges by making the transformation parameter finite and field-dependent, as one considers a sequence of infinitesimal field-dependent BRST transformations (for a numeric parameter κ) with some applications [31–44]. Another way to consider a finite field-dependent parameter in Yang–Mills theories was inspired by a research devoted to the so-called *soft BRST symmetry breaking* problem [45], with reference to the Gribov problem [46], which involves the Zwanziger proposal [47] for a horizon functional joined additively to a BRST-invariant quantum ac-

tion. In fact, the horizon functional in R_ξ -gauges with small ξ was found explicitly [45] (see Eq. (5.20) therein) by using field-dependent BRST transformations with a small odd-valued parameter, which was then extended to be finite [48]. The case of finite field-dependent BRST transformations for general gauge theories was considered in [49], whereas for BRST-antiBRST symmetry [50–52] in [53, 54], with the original calculation algorithm for functional Jacobians (for a comparative analysis of BRST symmetry, see [55]).

At the same time, analogous properties of space-time SUSY transformations (with Grassman-odd parameters) have never been generalized. Therefore, in spite of the fact that BRST transformations are realized in an extended field space with the initial classical, as well as the ghost, antighost and Nakanishi–Lautrup fields and are reminiscent of gauge transformations, a similar application of SUSY transformations in the path integral with FD Grassman-odd parameters to field-theoretical models (without auxiliary field variables introduced via the Faddeev–Popov prescription [56]) provides us with an opportunity to apply the above research to the study of an influence of SUSY transformations on the quantum action structure.

In this paper, we consider a generalization of SUSY transformations to the case of m -parametric Lie superalgebra with the transformation parameters being finite and field-dependent. In this way, the resulting transformations remain a symmetry of the supersymmetric action. Under generalized SUSY transformations with arbitrary field-dependent parameters the functional measure, however, is not invariant. This leads to a non-trivial Jacobian for the functional measure and therefore to a modification of the quantum action by non-quadratic terms being a SUSY-exact contribution. For some choices of parameters, the generalized SUSY transformations amount a precise change in the exponent action. We illustrate these results by the example of a free toy model with $(1, 1)$ physical degrees of freedom, describing a supersymmetric harmonic oscillator with the generalized $N = 1$ and $N = 2$ SUSY transformations. In such a theory, the interaction terms emerge naturally in the functional integral under thus generalized SUSY with specific parameters.

The paper is organized as follows. In Section II, we study the generalized SUSY transformations with m Grassman-odd parameters for a general supersymmetric invariant theory, calculate the corresponding Jacobian of the change of variables, derive the standard and modified Ward identities and classify the interactions. In Section III, we illustrate the example of generalized SUSY transformations by a supersymmetric harmonic oscillator with $(1, 1)$ degrees of freedom, in such a way that the trivial interaction terms are produced by generalized $N = 1$, $N = 2$ SUSY transformations from the functional measure. Finally, we summarize the results in Conclusions.

We use the DeWitt condensed notation and the conventions of [30, 49, 53], e.g., $\varepsilon(F)$ denotes the value of Grassmann parity of a quantity F .

II. GENERALIZED SUSY TRANSFORMATIONS

Here, we investigate a finite field-dependent SUSY (FSUSY) transformation for general supersymmetric invariant theories (following the techniques developed in both [30] and [49, 53]). To this end, we first define a SUSY transformation with infinitesimal Grassmann-odd constant parameters ϵ^α , $\alpha = 1, \dots, m$, $\varepsilon(\epsilon^\alpha) = 1$, leaving invariant an action $S(q)$ of generic variables q^i , $i = 1, \dots, n$, $n = (n_+, n_-)$, $\varepsilon(q^i) = \varepsilon_i$:

$$\delta_\epsilon q^i = \mathcal{R}_\alpha^i(q) \epsilon^\alpha = q^i \overleftarrow{s}_\alpha \epsilon^\alpha : \quad S(q + \delta_\epsilon q) = S(q) + o(\epsilon) \iff S(q) \overleftarrow{\partial}_i \mathcal{R}_\alpha^i(q) = S(q) \overleftarrow{s}_\alpha = 0, \quad (1)$$

where $\overleftarrow{\partial}_i \equiv \frac{\overleftarrow{\partial}}{\partial q^i}$ and $\mathcal{R}_\alpha^i(q)$, \overleftarrow{s}_α , $\varepsilon(\mathcal{R}_\alpha^i, \overleftarrow{s}_\alpha) = (\varepsilon_i + 1, 1)$ are, respectively, the generators of SUSY transformations acting on the variables q^i and those acting on functionals $F(q)$. We suppose that the generators of SUSY transformations satisfy Abelian anticommutator relations:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = 0 \iff \{\overleftarrow{s}_\alpha, \overleftarrow{s}_\beta\} = 0. \quad (2)$$

The group transformations with finite parameters, ϵ^α , $q^i \rightarrow q'^i = q^i(q|\epsilon)$, may be restored by two equivalent ways from the Lie equations and from the requirement for any \overleftarrow{s}_α -closed functional $F(q)$ to

be invariant with respect to right group transformations:

$$q^i(q|\epsilon)\overleftarrow{\partial}_\alpha = q^i(q|\epsilon)\overleftarrow{s}_\alpha \left(\text{for } \overleftarrow{\partial}_\alpha \equiv \frac{\overleftarrow{\partial}}{\partial\epsilon^\alpha} \right) \iff F(q^i(q|\epsilon)) = F(q). \quad (3)$$

For a t -rescaled argument $\epsilon^\alpha \rightarrow t\epsilon^\alpha$ of $q^i(q|t\epsilon)$, the form of Lie equations is equivalent to (3) with a formal solution for constant ϵ^α

$$\frac{d}{dt}q^i(q|t\epsilon) = q^i(q|t\epsilon)\overleftarrow{s}_\alpha\epsilon^\alpha \implies q^i(q; t\epsilon) = q^i \exp\{t\overleftarrow{s}_\alpha\epsilon^\alpha\}, \quad (4)$$

so that the set of finite transformations forms an Abelian group $G = \{g(\epsilon) : g(\epsilon) = \exp\{t\overleftarrow{s}_\alpha\epsilon^\alpha\}, g(\epsilon_1)g(\epsilon_2) = g(\epsilon_2)g(\epsilon_1)\}$, for constant ϵ^α . For field-dependent $\epsilon^\alpha = \epsilon^\alpha(q)$ having no explicit dependence on space-time coordinates x^μ , $\partial_\mu\epsilon^\alpha(q) = 0$, the set of algebraic elements $\mathcal{G} = \{\tilde{g}_{lin}(\epsilon(q)) := 1 + \overleftarrow{s}_\alpha\epsilon^\alpha(q)\}$ forms a non-linear algebra which corresponds to a set of formal group-like finite elements:

$$\tilde{G} = \left\{ \tilde{g}(\epsilon(q)) : \tilde{g} = 1 + \sum_{i=1}^m \frac{1}{i!} \left[\prod_{k=1}^i (\overleftarrow{s}_{\alpha_k}) \prod_{k=1}^i \epsilon^{\alpha_{k+1-i}}(q) \right] \right\}, \quad (5)$$

obtained as solutions for $F(q^i(q|\epsilon(q))) = F(q)$ in (3) with finite FD $\epsilon^\alpha(q)$, as in [53]. Note that in the case $m = 1, 2$ we have representations of finite BRST and BRST-antiBRST group (4) and group-like elements (5) [49, 53].

We refer to SUSY transformations generalized in such a way as FSUSY transformations. Another way to derive FSUSY transformations for an $N = 1$ -parametric subset from G , i.e., for $m = 1$ can be done by rendering the infinitesimal parameter $\epsilon^1 \equiv \epsilon$ field-dependent through a continuous interpolation of an arbitrary parameter κ ($0 \leq \kappa \leq 1$), following Ref. [30]: $(q^i(\kappa = 0); q^i(\kappa = 1)) = (q^i; q^i(q|\epsilon))$.

An infinitesimal field-dependent SUSY transformation can be defined as

$$dq^i(\kappa) = \mathcal{R}^i(q)\epsilon'(q(\kappa))d\kappa, \quad (6)$$

where $\epsilon'(q(\kappa))d\kappa$ is an infinitesimal field-dependent parameter. An FSUSY transformation with a finite field-dependent parameter can now be constructed by integrating such an infinitesimal transformation from $\kappa = 0$ to $\kappa = 1$, as follows:

$$q^i(q|\epsilon) \equiv q^i + \mathcal{R}^i(q)\epsilon(q), \text{ where } \epsilon(q) = \int_0^1 d\kappa \epsilon'(q(q|\kappa)). \quad (7)$$

Note that in the case $m > 1$ it is impossible to restore FSUSY transformations in this way explicitly using (6) by a simple integration over an auxiliary κ (for details, see [30]).

Turning to FSUSY transformations with m odd parameters, we can see that such transformations remain a symmetry of the supersymmetric action, which is supposed to describe a non-degenerate non-gauge theory, whereas the path integral measure will not be invariant under such transformations and thereby will lead to a non-trivial Jacobian for a corresponding change of variables in the generating functional $Z(J)$ of Green's functions with external sources, J_i ($\varepsilon(J_i) = \varepsilon_i$) and in the path integral $Z(0) = Z_0$:

$$Z(J) = \int \mathcal{D}q \exp \left\{ \frac{i}{\hbar} [S(q) + J_i q^i] \right\}, \quad Z_0 = \int \mathcal{I}_q \text{ with } \mathcal{I}_{q\tilde{g}(\epsilon)} = J(q)\mathcal{I}_q, \quad (8)$$

$$\text{where } J(q) = \text{Sdet} \left\| q^i(q|\epsilon(q)) \overleftarrow{\partial}_j \right\| = \exp \{ \text{Str} \ln (\delta_j^i + M_j^i(q, \epsilon)) \} \text{ for } M_j^i(q, \epsilon) = \Delta q^i(q|\epsilon) \overleftarrow{\partial}_j, \quad (9)$$

which vanishes when $\epsilon^\alpha = \text{const}$, $M_j^i(q, \epsilon)|_{\epsilon=\text{const}} = 0$. The Jacobian can be calculated explicitly, following the recipe [49, 53, 54], and also by using the Green function method [57]. The latter approach, using t -rescaled parameteres $t\epsilon^\alpha$ (4) and the inverse (formal) transformations $\tilde{g}^{-1}(\epsilon(q))$,

$$q^i(q|t\epsilon)\tilde{g}^{-1}(\epsilon(q)) = q^i \implies q^i(q'|t\epsilon)\overleftarrow{\partial}_\alpha = -tq^i\overleftarrow{s}_\alpha, \quad (10)$$

assumes that the representation for $\ln J(q)$ given by (9) reads

$$\ln J(q) = \text{sTr} \ln \left(\delta_j^i - q^i(q'|\epsilon) \overleftarrow{\partial}_\alpha (\epsilon^\alpha \overleftarrow{\partial}_j) \right) \implies \frac{d}{dt} \ln J(q) = -\text{tr}_G ([e + tm]^{-1} m), \quad m_\beta^\alpha = \epsilon^\alpha \overleftarrow{s}_\beta, \quad (11)$$

where $(e)_\beta^\alpha$ and tr_G denote δ_β^α and trace over matrix G indices. In deriving (11), we have used the fact that in differentiating with respect to t , the first of the above equalities reads

$$G_j^i [q^j \overleftarrow{s}_\alpha] [\epsilon^\alpha(q) \overleftarrow{\partial}_i] (-1)^{\varepsilon_i} \text{ and follows from } G_j^i + t[q^i \overleftarrow{s}_\alpha] [\epsilon^\alpha(q) \overleftarrow{\partial}_k] G_j^k = \delta_j^i. \quad (12)$$

From the latter representation, we find

$$\epsilon^\alpha(q) \overleftarrow{\partial}_k G_j^k = ([e + tm]^{-1})_\beta^\alpha (\epsilon^\beta(q) \overleftarrow{\partial}_j), \quad (13)$$

so that after substitution in the first term of (12) we get the representation for the last quantity in (11), which after integration leads to the final result for the Jacobian (because $\ln J(q(0)) = 0$)

$$J(q(\epsilon)) = \exp \left\{ -\text{tr}_G \ln ([e + m]) \right\}. \quad (14)$$

The Jacobian for $m = 1, 2$ is reduced to already known Jacobians for $N = 1, 2$ finite FD BRST transformations with nilpotent, $\overleftarrow{s}, \overleftarrow{s}_a$, $a = 1, 2$. For functionally-independent FD $\epsilon_\alpha(q)$ the Jacobian is not \overleftarrow{s}_α -closed, in general, whereas for \overleftarrow{s}_α -potential (thereby functionally-dependent) parameters

$$\hat{\epsilon}^\alpha(q) = \frac{1}{(m-1)!} \Lambda(q) \varepsilon^{\alpha\alpha_1 \dots \alpha_{m-1}} \overleftarrow{s}_{\alpha_1} \dots \overleftarrow{s}_{\alpha_{m-1}}, \text{ for } \varepsilon^{12 \dots m} = 1 \text{ and } \varepsilon^{\alpha_0 \alpha_1 \dots \alpha_{m-1}} \varepsilon_{\alpha_{m-1} \dots \alpha_1 \alpha_0} = m! \quad (15)$$

with an arbitrary potential functional, $\Lambda(q)$, $\varepsilon(\Lambda) = m$, and totally antisymmetric tensors $\varepsilon^{\alpha_0 \alpha_1 \dots \alpha_{m-1}}$, $\varepsilon_{\alpha_{m-1} \dots \alpha_1 \alpha_0}$ the Jacobian is \overleftarrow{s}_α -closed.

Due to the equivalence theorem [58], the change of variables in $Z(J)$ and in the path integral Z_0 generated by FSUSY transformations (in terms of the integrand)

$$\mathcal{I}_{q\tilde{g}(\epsilon)} = J(q) \mathcal{I}_q = Dq \exp \left\{ \frac{i}{\hbar} [S(q) + i\hbar \text{tr}_G \ln ([e + m])] \right\} = Dq \exp \left\{ \frac{i}{\hbar} [S(q) + S_1(q, \epsilon(q))] \right\}, \quad (16)$$

leads to the same quantum theory, $Z_0 = Z_\epsilon$, with the same conventional S -matrix. At the same time, a representation for the transformed action, $S(q, \epsilon(q)) = S(q) + S_1(q, \epsilon(q))$, should be supersymmetrically invariant: $S(q, \epsilon(q)) \overleftarrow{s}_\alpha = 0$. FSUSY transformations which satisfy the above must obey the condition

$$S_1(q, \epsilon(q)) \overleftarrow{s}_\alpha = i\hbar \text{tr}_G \ln ([e + m]) \overleftarrow{s}_\alpha = 0, \quad (17)$$

In particular, $N = m$ FSUSY transformations $\tilde{g}(\hat{\epsilon}^\alpha(q))$ with FD parameters (15) for any potential $\Lambda(q)$ satisfy the condition (17).

Therefore, only *trivial interactions* $U_{tr}(q)$ can be generated (locally) by FSUSY transformations in the path integral, which are characterized by the condition $U_{tr}(q) \overleftarrow{s}_\alpha = 0$, whereas the *non-trivial interactions* $U(q)$ which lead to a different S -matrix should satisfy the requirement

$$U(q) \overleftarrow{s}_\alpha = 0 : \quad U(q) \neq V^\alpha(q) \overleftarrow{s}_\alpha \forall V^\alpha(q). \quad (18)$$

For $m = 1$, $m = 2$ FSUSY transformations, the corresponding Jacobians (for functionally dependent $\epsilon_a = \Lambda \overleftarrow{s}_a$, $(\epsilon_a, \overleftarrow{s}^a) = (\varepsilon_{ab} \epsilon^b, \varepsilon^{ab} \overleftarrow{s}_b)$ with antisymmetric $\varepsilon_{ab} = -\varepsilon_{ba}$ and ε^{ab} : $\varepsilon^{ab} \varepsilon_{bc} = \delta_c^a$, under the normalization $\varepsilon^{12} = 1$)

$$J_{(1)}(q(\epsilon)) = (1 + \epsilon \overleftarrow{s})^{-1} \Rightarrow J_{(1)} \overleftarrow{s} = 0 \text{ and } J_{(2)}(q(\Lambda \overleftarrow{s}_a)) = \left(1 + \frac{1}{2} \Lambda \overleftarrow{s}_a \overleftarrow{s}^a \right)^{-2} \Rightarrow J_{(2)} \overleftarrow{s}_a = 0, \quad (19)$$

lead only to trivial interactions. The invariance of the integrand \mathcal{I}_q (8) with respect to FSUSY with constant parameters ϵ^α leads to the presence of Ward identities for $Z(J)$:

$$J_i \langle q^i \overleftarrow{s}_\alpha \rangle_J = 0 \text{ where } \langle A(q) \rangle_J = Z^{-1}(J) \int \mathcal{D}q A(q) \exp \left\{ \frac{i}{\hbar} [S(q) + J_i q^i] \right\}, \quad \langle 1 \rangle_J = 1, \quad (20)$$

with a source-dependent average expectation value for a certain functional $A(q)$ corresponding to a given action $S(q)$. In turn, the property (16), with account taken of (14), for FD FSUSY transformations means the presence of a so-called *modified Ward identity* depending on FD parameters $\epsilon(q)$:

$$\left\langle \exp \left\{ \frac{i}{\hbar} J_i q^i \sum_{i=1}^m \frac{1}{i!} \left[\prod_{k=1}^i (\overleftarrow{s}_{\alpha_k}) \prod_{k=1}^i \epsilon^{\alpha_{k+1-i}}(q) \right] \right\} \exp \left\{ -\text{tr}_G \ln([e + m]) \right\} \right\rangle_J = 1, \quad (21)$$

(for $m_\beta^\alpha = \epsilon^\alpha(q) \overleftarrow{s}_\beta$) which reduces to (20) for constant ϵ^α .

In the case $m = 1$ (but not $m > 1$) FSUSY may also be considered by evaluation of the Jacobian according to [30], restricted by an infinitesimal FD parameter $\epsilon'(q(\kappa))$ according to the change of variables $q^i(\kappa) \rightarrow q^i(\kappa + d\kappa)$ with Jacobian $J(\kappa)$ (9):

$$\mathcal{D}q(\kappa + d\kappa) = J(\kappa) \mathcal{D}q(\kappa) \stackrel{def}{=} \mathcal{D}q(\kappa) \exp \left[-(-1)^i \mathcal{R}^i(q)(\kappa) (\epsilon'(q(\kappa)) \overleftarrow{\partial}_i^{(q(\kappa))}) \right], \quad (22)$$

$$\text{for } J(\kappa) = 1 - \left[(-1)^i \mathcal{R}^i(q)(\kappa) (\epsilon'(q(\kappa)) \overleftarrow{\partial}_i^{(q(\kappa))}) \right]. \quad (23)$$

As we suppose that after a change of variables generated by FSUSY transformations $q^i \rightarrow q^i(\kappa)$ the supersymmetric action $S(q)$ also changes to $S(q) + S_1(\kappa)$ with a local functional $S_1(q)$ vanishing at $\kappa = 0$, the functional equation must hold

$$\int \mathcal{D}q(\kappa) \left[\frac{d}{d\kappa} \ln J(\kappa) - \frac{i}{\hbar} \frac{d}{d\kappa} S_1(q(\kappa)) \right] \exp \left\{ \frac{i}{\hbar} [S(q) + S_1(q(\kappa))] \right\}. \quad (24)$$

The necessary condition that equation (24) be solvable is $S_1(q(\kappa)) \overleftarrow{s} = 0$, i.e., the addition to supersymmetric action must also be supersymmetric. Once again, FSUSY transformations with appropriate parameters ϵ change a supersymmetric action S_{susy} to a new effective action $S_{susy} + S_1(\kappa = 1)$ within functional integration.

Note that one can perform a similar analysis in the case of $N = 1$ SUSY transformations with parameters $\bar{\epsilon}$ and the result will be the same. The only difference is that the parameter ϵ will be replaced everywhere by $\bar{\epsilon}$ and the generator $R_1^i(q)$ will be replaced by $R_2^i(q)$.

III. SUPERSYMMETRIC HARMONIC OSCILLATOR

In this section, we analyse an $N = 1$ supersymmetric free toy model with $(1, 1)$ physical degrees of freedom described by one bosonic x and two fermionic $\psi, \bar{\psi}$ coordinates: collectively, $q^i = (x, \psi, \bar{\psi})$, $i = 1, \dots, n; n = (1, 2)$, from the generalized SUSY perspectives. Here, we find that (trivial) interaction terms for such a supersymmetric model emerge naturally through the Jacobian of functional measure. Let us start by writing the classical action $S(q)$ for a supersymmetric harmonic oscillator:

$$S = \int_{t_{in}}^{t_{out}} dt \left[\frac{1}{2} \dot{x}^2 - \frac{1}{2} \omega^2 x^2 + i \bar{\psi} \dot{\psi} - \omega \bar{\psi} \psi \right]. \quad (25)$$

This action refers to a free toy model with $(1, 1)$ physical degrees of freedom, formally due to the presence of second-class constraints for ψ . Here, we pass to dimensionless quantities, so that, for convenience, the mass is $m = 1$ for the bosonic part. The action is invariant up to a total time derivative with respect to an $N = 1$ subalgebra of the total SUSY superalgebra with parameter $\epsilon^1 = \epsilon$ ($m = 1$),

$$\delta_\epsilon [x, \psi, \bar{\psi}] = \frac{1}{\sqrt{2}} [\bar{\psi}, -i\dot{x} - \omega x, 0] \epsilon \equiv [x, \psi, \bar{\psi}] \overleftarrow{s} \epsilon, \quad (26)$$

and also with respect to an $N = 1$ subalgebra with an odd parameter $\epsilon^2 = \bar{\epsilon}$,

$$\delta_{\bar{\epsilon}}[x, \psi, \bar{\psi}] = -\frac{1}{\sqrt{2}}[\psi\bar{\epsilon}, 0, -i\dot{x} + \omega x]\bar{\epsilon} \equiv [x, \psi, \bar{\psi}] \overleftarrow{s} \bar{\epsilon}, \quad (27)$$

which relates with the above subalgebra by means of complex conjugation, $\delta_{\bar{\epsilon}}[x, \psi, \bar{\psi}] = (\delta_{\epsilon}[x, \bar{\psi}, \psi])^*$, determined by the rule:

$$(x, \psi, \bar{\psi}, \epsilon, \bar{\epsilon})^* = (x, \bar{\psi}, \psi, \bar{\epsilon}, \epsilon) \text{ and } (ab)^* = b^*a^* \text{ for } a, b \in \{x, \psi, \bar{\psi}, \epsilon, \bar{\epsilon}\}. \quad (28)$$

$N = 2$ SUSY algebraic transformations are determined by the identification $\epsilon^a = (\epsilon, \bar{\epsilon})$, $\psi^a = (\psi, \bar{\psi})$:

$$\delta_{\epsilon_a}[x, \psi^b] = [x, \psi^b] \overleftarrow{s}_a \epsilon^a \equiv R_a^i(q) \epsilon^a, \text{ for } R_a^i = \frac{1}{\sqrt{2}}[\varepsilon_{ca} \psi^c, [(-1)^b i\dot{x} - \omega x] \delta_a^b], \quad (29)$$

whereas $N = 2$ FSUSY transformations form an Abelian group $\{g(\epsilon^a) = \exp\{\overleftarrow{s}_a \epsilon^a\}\}$, and quadratic terms in powers of $(\epsilon)^2 = \epsilon_a \epsilon^a = 2\epsilon\bar{\epsilon}$ together with finite transformations realized on q^i are

$$[x, \psi^b] \overleftarrow{s}^a \overleftarrow{s}_a (\epsilon)^2 = \left[\omega x, \frac{1}{2}((-1)^b i\dot{x} - \omega) \psi^b \right] (\epsilon)^2 \implies \Delta_{\epsilon^a}[x, \psi^b] = \delta_{\epsilon_a}[x, \psi^b] + \frac{1}{4}[x, \psi^b] \overleftarrow{s}^a \overleftarrow{s}_a (\epsilon)^2, \quad (30)$$

then $S([x, \psi^b]g(\epsilon^a)) = S(x, \psi^b)$ for arbitrary finite FD ϵ^a . SUSY invariant interaction terms (with respect to algebraic transformations) may be given by (for $n > 1$) polynomial in $x, \psi, \bar{\psi}$

$$\begin{aligned} S_{int} &= \int_{t_{in}}^{t_{out}} dt \sum_{n=2}^M g_n \left[-\frac{n}{\sqrt{2}} x^{n-1} \bar{\psi} \psi - \frac{i}{\sqrt{2}} x^n \dot{x} - \frac{1}{\sqrt{2}} \omega x^{n+1} \right] = \int_{t_{in}}^{t_{out}} dt \sum_{n=2}^M g_n [\delta_{\epsilon}(x^n \psi)] \overleftarrow{\partial}_{\epsilon}, \\ &= - \int_{t_{in}}^{t_{out}} dt \sum_{n=2}^M g_n \left[\frac{\sqrt{2}}{(n+1)} \delta_{\epsilon} \delta_{\bar{\epsilon}}(x^{n+1}) \right] \overleftarrow{\partial}_{\epsilon} \overleftarrow{\partial}_{\bar{\epsilon}} = - \int_{t_{in}}^{t_{out}} dt \sum_{n=2}^M g_n \left[\frac{\sqrt{2}}{2(n+1)} (x^{n+1}) \overleftarrow{s}^a \overleftarrow{s}_a \right], \end{aligned} \quad (31)$$

with some coupling constants g_n providing a correct dimension of the action. The interaction (31) appears trivial, due to definition (18). Then, the full action incorporating interaction, $S_{full} = S + S_{int}$, is invariant under $N = 1$ FSUSY transformations given by (26) and (27), as well as under $N = 2$ FSUSY transformations (30).

The generators of SUSY transformations (26), (27) and (29) can be presented from a standard SUSY representation using the supercommutator $[\ , \]$ for equal times:

$$\delta_{\epsilon} q^i = i[q^i, Q]\epsilon, \quad \delta_{\bar{\epsilon}} q^i = i[q^i, \bar{Q}]\bar{\epsilon}, \quad (32)$$

with an explicit realization of supercharges

$$(Q, \bar{Q}) = \frac{1}{\sqrt{2}} \left(\overleftarrow{\partial}_x \bar{\psi} + \overleftarrow{\partial}_{\psi} [i\dot{x} + \omega x], \overleftarrow{\partial}_x \psi + \overleftarrow{\partial}_{\bar{\psi}} [-i\dot{x} + \omega x] \right). \quad (33)$$

which is nothing else than s and \bar{s} , respectively, satisfying the algebra (2).

A. Generalized SUSY transformations and Jacobians

Following Section II, we generalize the SUSY transformations (26), (27) and (30) by making the transformation parameters finite and field-dependent:

$$(\delta_{\epsilon}, \delta_{\bar{\epsilon}}) q^i = q^i \left(\overleftarrow{s} \epsilon(q), \overleftarrow{s} \bar{\epsilon}(q) \right) \text{ and } \Delta_{\epsilon^a} q^i = q^i \left(\overleftarrow{s}_a \epsilon^a(q) + \frac{1}{4} \overleftarrow{s}^a \overleftarrow{s}_a (\epsilon(q))^2 \right) \quad (34)$$

Corresponding to $N = 1$ and $N = 2$ FSUSY transformations, the Jacobians of a change of variables in the path integral (8) in question are given by (19):

$$J(q(\epsilon)) = (1 + \epsilon \overleftarrow{s})^{-1} \text{ and } J(q(\Lambda \overleftarrow{s}_a)) = \left(1 + \frac{1}{2} \Lambda \overleftarrow{s}_a \overleftarrow{s}^a\right)^{-2}, \text{ for } \epsilon_a(q) = \Lambda(q) \overleftarrow{s}_a. \quad (35)$$

where $\Lambda(q)$ is an arbitrary bosonic functional. When considering the recipe [30], the finite FD parameter $\epsilon(q)$ presented in terms of an infinitesimal $\epsilon'(q(\kappa))$:

$$\epsilon(q) = \int_0^1 d\kappa \epsilon'(q(\kappa)) \text{ and } \bar{\epsilon}(q) = \int_0^1 d\kappa \bar{\epsilon}'(q(\kappa)) \quad (36)$$

represent arbitrary finite FD SUSY parameters. The Jacobian of both $N = 1$ SUSY transformations can be calculated using (22).

This shows that the interactions terms (31) may be generated by $N = 1$ and $N = 2$ FSUSY transformations with appropriate parameters.

B. Generating the interaction terms

To find an explicit finite FD parameter $\epsilon(q)$ for $N = 1$ SUSY transformations which generates the trivial interaction terms (31), we consider the functional equation

$$Z_0 = Z_{int} \text{ where } Z_{int} = \int \mathcal{D}q \exp \left\{ \frac{i}{\hbar} [S(q) + S_{int}(q)] \right\}, \quad (37)$$

and Z_0 is determined in (8). Making a change of variables in the integrand of Z_0 generated by FSUSY, we obtain an equation with accuracy up to a total functional derivative:

$$i\hbar \ln J(q(\epsilon)) + S_{int}(q) = 0 \iff i\hbar \ln(1 + \epsilon(q) \overleftarrow{s}) = \int_{t_{in}}^{t_{out}} dt \sum_{n=2}^M g_n [x^n \psi] \overleftarrow{s}, \quad (38)$$

which we call a *compensation equation*. Because both parts of the compensation equation are s -exact, we determine the unknown $\epsilon(q)$ in terms of S_{int}

$$\epsilon(q|S_{int}) = \frac{i}{\hbar} g(y) \int_{t_{in}}^{t_{out}} dt \sum_{n=2}^M g_n [x^n \psi], \text{ for } g(y) = \frac{1 - \exp\{y\}}{y}, \quad y \equiv \frac{i}{\hbar} S_{int}. \quad (39)$$

Vice-versa, considering equation (38) for some unknown interaction, we can always construct a trivial interaction $S_{int} = U(q) \overleftarrow{s}$ for any $N = 1$ FSUSY transformation with a given $\epsilon(q)$:

$$U(q|\epsilon) = \frac{\hbar}{i} \left[\sum_{n=1} \frac{(-1)^{n-1}}{n} (\epsilon \overleftarrow{s})^{n-1} \right] \epsilon. \quad (40)$$

The same can be done for an $N = 1$ FSUSY with $\bar{\epsilon}(q)$ concerning a one-to-one correspondence among trivial interactions, represented as $\bar{U}(q) \overleftarrow{s}$, $\epsilon(\bar{U}(q)) = 1$, and a set of respective $N = 1$ FSUSY transformations.

Concerning the case of $N = 2$ FSUSY transformations with $\epsilon^a(q)$, $a = 1, 2$, the generation of trivial $N = 2$ supersymmetric interactions is the same for functionally-dependent $\epsilon^a(q) = \Lambda \overleftarrow{s}^a$. The corresponding compensation equation to provide (37) and its solution for a given interaction (31) with bosonic the potential $U_2(q) = \sum_{n=2}^M g_n \frac{\sqrt{2}}{2(n+1)} x^{n+1}$: $S_{int} = U_2(q) \overleftarrow{s}^a \overleftarrow{s}_a$,

$$i\hbar \ln J(q(\epsilon)) + S_{int}(q) = 0 \iff i\hbar \ln(1 + \frac{1}{2} \Lambda(q) \overleftarrow{s}_a \overleftarrow{s}^a) = -U_2(q) \overleftarrow{s}^a \overleftarrow{s}_a, \quad (41)$$

$$\Lambda(q|U_2) \overleftarrow{s}_a = \epsilon_a(q|U_2) = \frac{i}{2\hbar} g(y) U_2 \overleftarrow{s}_a, \quad \Lambda(q|U_2) = \frac{i}{2\hbar} g(y) U_2, \text{ for } y \equiv (i/4\hbar) U_2 \overleftarrow{s}^a \overleftarrow{s}_a. \quad (42)$$

Conversely, for an unknown interaction we can always construct a trivial interaction, $S_{int} = U_2(q) \overleftarrow{s}^a \overleftarrow{s}_a$, for any $N = 2$ FSUSY transformation with a given $\epsilon^a(q) = \Lambda \overleftarrow{s}^a$:

$$U_2(q|\epsilon_a) = 4i\hbar \left[\sum_{n=1} \frac{(-1)^{n-1}}{2^n n} (\Lambda \overleftarrow{s}^a \overleftarrow{s}_a)^{n-1} \Lambda \right]. \quad (43)$$

Therefore, if the trivial interaction S_{tr} is given by $S_{tr} = U \overleftarrow{s} = \bar{U} \overleftarrow{s} = U_2 \overleftarrow{s}^a \overleftarrow{s}_a$ then it can be generated (or removed from the initial action) by any $N = 1, 2$ FSUSY transformation with respective $\epsilon(q), \bar{\epsilon}(q), \epsilon^a(q) = \Lambda \overleftarrow{s}^a$.

Omitting the details of a similar application of $N = 1$ FSUSY transformations in the form (36) to derive interaction (31), we stress that solving the problem amounts to calculating the Jacobian $J(\kappa)$ in equation (22). To find an unknown $S_1(q(\kappa), \kappa)$, we consider an infinitesimal FD parameter in the form

$$\epsilon' = - \int_{t_{in}}^{t_{out}} dt \sum_{n=2}^M (g_n x^n \psi). \quad (44)$$

We then choose the following ansatz for S_1 :

$$S_1(q(\kappa), \kappa) = - \int_{t_{in}}^{t_{out}} dt \sum_{n=2}^M g_n (\chi_1(\kappa) x^{n-1} \bar{\psi} \psi + \chi_2(\kappa) x^n \dot{x} + \chi_3(\kappa) x^{n+1}), \quad (45)$$

where $\chi_i, i = 1, 2, 3$ are constant κ -dependent parameters satisfying the condition $\chi_i(\kappa = 0) = 0$. From equation (24), we derive the following differential (in κ) equations:

$$\sqrt{2}\chi'_1 - n = 0, \quad \sqrt{2}\chi'_2 - \iota = 0, \quad \sqrt{2}\chi'_3 - \omega = 0, \quad (46)$$

whose obvious solution (as one integrates from 0 to κ)

$$(\chi_1, \chi_2, \chi_3) = \left(\frac{n}{\sqrt{2}}\kappa, \frac{\iota}{\sqrt{2}}\kappa, \frac{1}{\sqrt{2}}\omega\kappa \right), \quad (47)$$

leads to an explicit form of $S_1(q(\kappa), \kappa)$, while $\kappa = 1$ leads to S_{int} (31).

IV. CONCLUSIONS

We have extended the results and ideas of our previous study [30, 49, 53] considering special Abelian SUSY transformations as a symmetry of a Lagrangian action with bosonic and fermionic degrees of freedom, which form a superalgebra with m Grassman-odd parameters. The SUSY invariance of the action for infinitesimal values of the parameters is restored to the case of finite values by solving the Lie equations. As a result, we have constructed, starting from a Lie superalgebra, a Lie supergroup [where exp-correspondence completely maps the Lie superalgebra to the Lie supergroup (4)] with each of its element being an invariance transformation of the supersymmetric action in powers of the Grassman-odd parameters. This construction generalizes the case of BRST ($m = 1$) and BRST-antiBRST ($m = 2$) finite transformations for gauge theories with a closed gauge algebra, including Yang-Mills theories. We have calculated the Jacobian of a change of variables in the path integral with a supersymmetric action, given by finite SUSY transformations with field-dependent parameters in (14), which contains as a partial case the Jacobians of formal BRST and BRST-antiBRST finite FD transformations. Because the set of FSUSY transformations satisfies the equivalence theorem [58] conditions, the addition from the functional measure in the path integral may modify the supersymmetric action by a Jacobian more than quadratic in powers of fields that still leads to the same conventional S -matrix. We have called such additions to the action trivial interactions. Non-trivial FSUSY invariant interactions cannot be generated by this receipt. It is shown that the presence of m -parametric FSUSY transformations leads to the presence of standard Ward identities for generating functionals of Green functions (20) corresponding to constant odd parameters, as well as to modified Ward identities (21) depending on FD finite odd parameters $\epsilon^\alpha(q)$.

We have illustrated these results by a simple free toy model with $(1,1)$ physical degrees of freedom describing a supersymmetric harmonic oscillator by a generalization of $N = 1$ and $N = 2$ SUSY transformations. It is shown that any trivial interaction can be completely generated from the functional measure by means of $N = 1$ and $N = 2$ FSUSY transformations respectively with FD parameter and functionally-dependent parameters.

The present research may be used to analyse the influence on the structure of a quantum action of real space-time SUSY transformations with FD parameters, which, however, do not form an Abelian superalgebra and contain, in addition to Q and \bar{Q} , also a Grassman-even generator of momenta, P^μ . At the same time, in the case of additional presence of gauge invariance for a supersymmetric action the problem of a joint consideration of FSUSY transformations and BRST or BRSTantiBRST transformations for a quantum action may prove to be a promising direction of research.

Acknowledgments The authors are grateful to P.Yu. Moshin for interest and discussions. The study of A.R. was supported by the RFBR grant No. 16-42-700702.

-
- [1] M. Green, J. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, (1987).
 - [2] S. Ferrara, D. Freedman and P. van Nieuwenhuizen, Phys. Rev. D 13, 3214 (1976).
 - [3] S. Deser and B. Zumino, Phys. Lett. B 62, 335 (1976).
 - [4] I. L. Buchbinder, E. A. Ivanov and I. B. Samsonov, arXiv:1603.02768 [hep-th].
 - [5] E. A. Ivanov, arXiv: 1604.01379 [hep-th].
 - [6] M. Vasiliev, Lect. Notes Phys. 892, 227 (2015).
 - [7] A. Fotopoulos and M. Tsulaia, Int. J. Mod. Phys. A. 24, 1 (2008), arXiv:0805.1346[hep-th].
 - [8] I. L. Buchbinder, V. A. Krykhtin and A. A. Reshetnyak, Nucl. Phys. B. 787, 211 (2007).
 - [9] A. A. Reshetnyak, Nucl. Phys. B. 869, 523 (2013).
 - [10] R. R. Metsaev, Phys. Lett. B. 720, 237 (2013).
 - [11] I. L. Buchbinder, T. V. Snegirev and Yu. M. Zinoviev, JHEP. 1510, 148 (2015).
 - [12] F. Cooper, A. Khare, and U. P. Sukhatme, *Supersymmetry in quantum mechanics*, World Scientific, Singapore (2001).
 - [13] E. Witten, Nucl. Phys. B 188, 513 (1981).
 - [14] F. Cooper and B. Freedman, Annls. Phys. 146, 262 (1983).
 - [15] P. Salomonson and J. van Holten, Nucl. Phys. B 196, 509 (1982).
 - [16] M. Bernstein and L. Brown, Phys. Rev. Lett. 52, 1933 (1984).
 - [17] P. Kumar, M. Ruiz-Altaba and B. S. Thomas, Phys. Rev. Lett. 57, 2749 (1986).
 - [18] F. Marchesoni, P. Sodano and M. Zannetti, Phys. Rev. Lett. 61, 1143 (1988).
 - [19] W.-Y. Keung, E. Kovacs and U. Sukhatme, Phys. Rev. Lett. 60, 41 (1988).
 - [20] C. Becchi, A. Rouet and R. Stora, Phys. Lett. B 52, 344 (1974); Annals Phys. 98, 287 (1976).
 - [21] I. V. Tyutin, Lebedev Inst. preprint. 39 (1975), arXiv: 0812.0580 [hep-th].
 - [22] C. Rupp, R. Scharf and K. Sibold, Nucl. Phys. B 612, 313 (2001).
 - [23] P. Van Nieuwenhuizen, arXiv: hep-th/0408179.
 - [24] de Witt, B., van Holten, J. W., Phys. Lett. B 79, 389 (1979).
 - [25] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B 102, 27 (1981).
 - [26] E. S. Fradkin and G. A. Vilkovisky, Phys. Lett. B 55, 224 (1975).
 - [27] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B 69, 309 (1977).
 - [28] M. Henneaux and C. Teitelboim, *Quantization of gauge systems*, Princeton, USA: Univ. Press (1992).
 - [29] S. Weinberg, *The quantum theory of fields, Vol-II: Modern applications*, Cambridge, UK Univ. Press (1996).
 - [30] S. D. Joglekar and B. P. Mandal, Phys. Rev. D 51, 1919 (1995).
 - [31] S. Upadhyay, Phys. Lett. B 723, 470 (2013); Annls. Phys. 344, 290 (2014).
 - [32] S. D. Joglekar and B. P. Mandal, Int. J. Mod. Phys. A 17, 1279 (2002).
 - [33] R. Banerjee and B. P. Mandal, Phys. Lett. B 488, 27 (2000).
 - [34] S. Upadhyay, S. K. Rai and B. P. Mandal, J. Math. Phys. 52, 022301 (2011).
 - [35] S. Upadhyay and B. P. Mandal, Phys. Lett. B 744, 231 (2015); arXiv:1407.2017 [hep-th]; Prog. Theor. Exp. Phys. 053B04 (2014); Eur. Phys. J. C 72, 2065 (2012); Annals of Physics 327, 2885 (2012); EPL 93, 31001 (2011); Mod. Phys. Lett. A 25, 3347 (2010).
 - [36] B. P. Mandal, S. K. Rai, and S. Upadhyay, EPL 92, 21001 (2010).

- [37] S. Upadhyay, M. K. Dwivedi and B. P. Mandal, *Int. J. Mod. Phys. A* 28, 1350033 (2013).
- [38] M. Faizal, B. P. Mandal and S. Upadhyay, *Phys. Lett. B* 721, 159 (2013).
- [39] R. Banerjee, B. Paul and S. Upadhyay, *Phys. Rev. D* 88, 065 019 (2013).
- [40] S. Upadhyay, *Annl. Phys.* 356, 299 (2015); *Phys. Lett. B* 740, 341 (2015); *EPL* 105, 21001 (2014); *Phys. Lett. B* 727, 293 (2013); *EPL* 104, 61001 (2013); arXiv:1308.0982 [hep-th].
- [41] R. Banerjee, B. Paul, S. Upadhyay, *Phys. Rev. D* 88 (2013) 065019.
- [42] S. Upadhyay and D. Das, *Phys. Lett. B* 733, 63 (2014).
- [43] R. Banerjee and S. Upadhyay, *Phys. Lett. B* 734, 369 (2014).
- [44] S. Upadhyay, *Annals. Phys.* 340, 110 (2014).
- [45] P. Lavrov, O. Lechtenfeld, A. Reshetnyak, *JHEP.* 1110, 043 (2011).
- [46] V.N. Gribov, *Nucl. Phys. B.* 139, 1 (1978).
- [47] D. Zwanziger, *Nucl. Phys. B.* 323, 513 (1989).
- [48] P. Lavrov and O. Lechtenfeld, *Phys. Lett. B.* 725, 382 (2013).
- [49] A. Reshetnyak, *Int. J. Mod. Phys. A* 29, 1450128 (2014).
- [50] G. Curci and R. Ferrari, *Phys. Lett. B.* 63, 91 (1976).
- [51] L. Alvarez-Gaume and L. Baulieu, *Nucl. Phys. B.* 212, 255 (1983).
- [52] V.P. Spiridonov, *Nucl. Phys. B.* 308, 527 (1988).
- [53] P. Yu. Moshin and A. A. Reshetnyak, *Nucl. Phys. B.* 888, 92 (2014); *Phys. Lett. B* 739, 110 (2014); *Int. J. Mod. Phys. A* 29, 1450159 (2014); *Int. J. Mod. Phys. A* 30, 1550021 (2014).
- [54] P. Yu. Moshin and A. A. Reshetnyak, arXiv: 1506.04660 [hep-th].
- [55] P. Yu. Moshin and A. A. Reshetnyak, arXiv: 1604.03027 [hep-th].
- [56] L.D. Faddeev and V.N. Popov, *Phys. Lett. B.* 25, 29 (1967).
- [57] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *Int. J. Mod. Phys. A.* 29, 1450166 (2014).
- [58] K.E. Kallosh and I.V. Tyutin, *Sov. J. Nucl. Phys.* 17, 98 (1973).